Composing Functions

Consider the two functions:

$$f(x) = 1 - x \quad \text{and} \quad g(x) = \frac{1}{x}$$

We can compose them in two ways:

$$f(g(x)) \quad \text{and} \quad g(f(x)).$$

We can go further and compose these two new functions with themselves, and also with the old ones, in a number of ways. Keep composing these functions with new ones as they are generated and figure out simplified formulae for them in terms of the variable x. (Don't forget to compose functions with themselves, like $f(f(x))$.) You might think that more and more new functions will be generated. Surprisingly only a finite number of new ones get generated by composition, even though there may be many different ways of composing f and g to get the same function. Remember that two very different looking formulae may represent the same function.

a. How many distinct functions are there, including f and g themselves?

b. List them.

c. How is each one composed from f and g?

d. How do you know that these are all there are?

e. For f(g(x)) what is the domain? For g(f(x)) what is the domain?